3-D Deformation of Newtonian droplets under simple shear investigated by experiment, numerical simulation and modelling.

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ABSTRACT

The deformation and break-up/coalescence of droplets in complex flow fields is encountered in many unit process operations in the chemical and pharmaceutical industries. In the present work, the three dimensional deformation of droplets is studied experimentally in simple shear flow and compared with numerical calculations and modelling efforts.

INTRODUCTION

The deformation and break-up of droplets in complex flow fields is encountered in many engineering applications such as mixing and dispersing processes. To manipulate and control such operations, rheological, interfacial, and dynamical properties of the multiphase fluid as well as their interaction have to be known.

In this work, we study experimentally the 3-D deformation of Newtonian droplets subjected to simple shear flow, and the results are compared with numerical calculations and modelling.

For this purpose, a computer-controlled parallel band apparatus equipped with two digital cameras records the time evolution of the sheared droplet and thus, analyzes digitally its shape[1].

Numerical simulations are performed to calculate the drop deformation in three-dimensional space, and are compared with experimental data. The simulations use a boundary integral method (BIM) to determine drop deformation from the mass and momentum balance equations[2]. Furthermore, a simple phenomenological model (DM) in the spirit of the Maffettone and Minale model is proposed to describe droplet deformation in homogeneous flow[3]. Whereas the BIM algorithm allows to compute the full shape of the droplets, the DM assumes an ellipsoidal droplet shape which is described in terms of the three semiaxes and the orientation angles with respect to the externally imposed flow field.

To describe droplet deformation, the viscosity of the dispersed fluid phase, η_d, the viscosity of the continuous fluid phase, η_c, and the interfacial tension, σ, are used. The capillary number, Ca, displays the ratio of viscous forces, which work to deform the drop, to interfacial tension, which works to restore the shape of the drop by Eq. 1.

$$Ca = \frac{\eta_c a \dot{\gamma}}{\sigma} \qquad (1)$$

where a is the radius of the undeformed drop and $\dot{\gamma}$ is the applied shear rate.

The interfacial tension is an experimental parameter which must be known in order to perform the numerical calculations. The application of the theory of Taylor[4] provides this information as a fitting parameter of the deformation parameter, $D = (L - B) / (L + B)$, as shown in Eq. 2.

$$D = \frac{19\lambda + 16}{16\lambda + 16} Ca \tag{2}$$

where L and B are the half-lengths of the sheared droplet in the flow direction and gradient direction, respectively, and λ is the viscosity ratio ($\lambda = \eta_d / \eta_c$).

MATERIALS AND METHODS

The measured systems are Newtonian, consisting of a solution of PEG-EtOH-H_2O as continuous phase (η_c = 290 mPa·s), and droplets of several silicon oils (Wacker, Germany) with the same density. The resulting viscosity ratios are λ = 0.165, 0.331, 3.35 and 16.7.

The experimental setup consists of a parallel band apparatus. The ribbons are metallic and spring-loaded to avoid any bending effect; their motion is computer-controlled independently through two motors. The flow profile developed in this device is linear and the shear-rate is simply obtained as the ratio between the relative speed at which the bands move and the distance between them.

The time sequence of the droplets behaviour is recorded with two CCD digital cameras (Sony DFW-V500, Japan), one placed along the z-axis (vorticity direction) and the second along the x-axis (flow direction). This experimental set-up allowed us to measure independently the three semiaxis of the ellipsoidal droplets.

The interfacial tension for the systems was previously obtained[1] using the described apparatus and utilizing Eq. 2, and its value found to be σ = 10.1±0.2 mN/m.

RESULTS AND DISCUSSION

Experiments on deformation of sheared droplets were carried out at different shear-rates for each viscosity ratio. Under the action of a homogeneous, steady shear flow, the droplet, initially at rest and immersed in the continuous fluid phase, progressively deforms into an ellipsoid and after a finite time it reaches a steady-state shape as long as flow conditions remain the same.

Fig. 1 shows this behaviour for λ = 16.7 and $\dot{\gamma}$ = 5.8 s^{-1}. The drop size evolution is quantified by L, B and W (half-length of the droplet along the vorticiy axis). For these conditions, the droplet which initially is a sphere (L, B and W equal to 1) becomes an ellipsoid after a finite time. Under the action of the flow, it extends in the flow direction ($L > a$) and it contracts in the gradient axis ($B < a$). Respect to the drop length in the vorticity direction, two possibilities are allowed, in a general case: a prolate configuration ($W < a$) or an oblate ellipsoid ($W < a$). For the system shown in Fig. 1, the first behaviour is found.

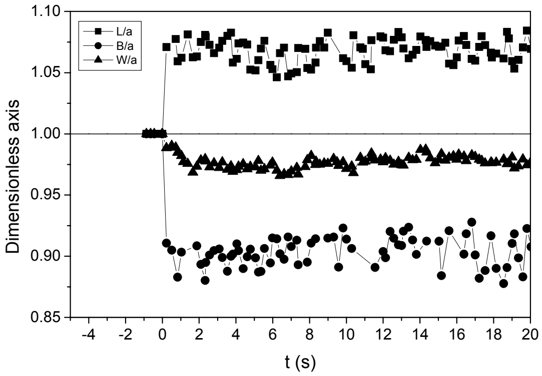

Figure 1. Dimensionless axis as a function of the experimental time, t. λ = 16.7, $\dot{\gamma}$ = 5.8 s^{-1}.

Analogous experiments were carried out for the fluid systems with other viscosity ratios and the steady normalized axes are plotted in Figs. 2a-c, as a function of the corresponding capillary numbers.

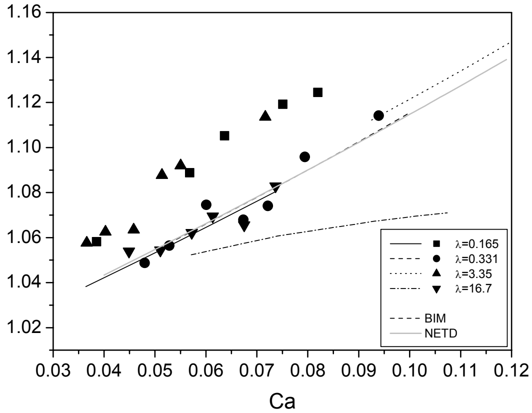

Figure 2a. Normalized major axis, *L / a*, as a function of the capillary number, *Ca*, for the different systems. Lines correspond to BIM and DM (gray line) calculations.

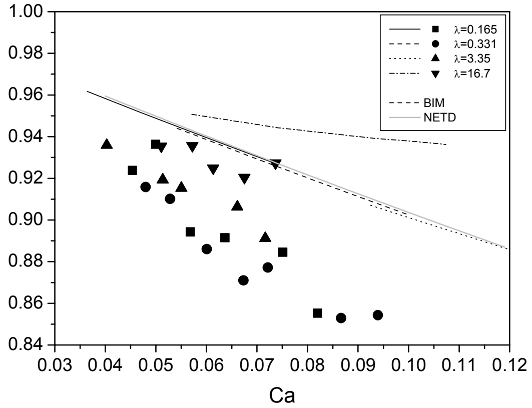

Figure 2b. Normalized minor axis, *B / a*.

Simulations of drop deformation were performed using the three-dimensional boundary integral method, BIM, and the dynamical model, DM, are shown in Figs. 2a-c as solid and dotted lines, respectively. Since the simulated drop is three-dimensional, we can completely determine the shape of the drop in all three directions. The viscosity ratios considered in the numerical study were those measured in the experiments. Both numerical methods agree with each other, founding practically the same results for the corresponding λ.

The numerical predictions agree qualitatively with the experimental data, for the dimensionless *L* and *B* axis. Quantitatively, both numerical methods underestimate the experimental results.

Respect to the behaviour of the droplet in the vorticity axis, we find experimentally that for droplets less viscous than the continuous matrix, $\lambda < 1$, it expands in the z-axis giving an oblate ellipsoid with $W/a > 1$. This behaviour is not captured by the simulation and the modelling methods which assume in all the cases a prolate configuration of the ellipsoidal droplet (Fig. 2c).

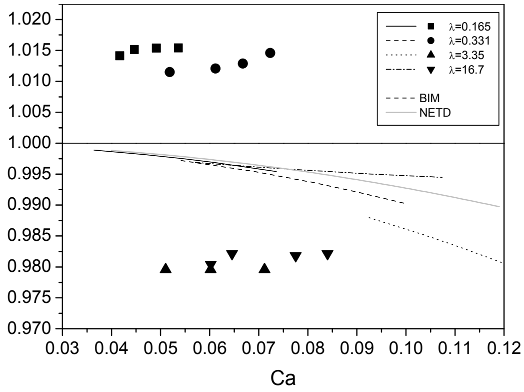

Figure 2c. Normalized vorticity axis, *W / a*.

On the other hand, when the droplet is more viscous than the fluid phase, $\lambda > 1$, the expected prolate configuration is observed, and then, $W/a < 1$, according to the predictions of the numerical simulations and modelling efforts.

SUMMARY

In the present investigation, we compare experimentally obtained droplet deformation with a Boundary Integral Method simulations and modelling efforts.

Both numerical methods agree with the experimental data and respect each other.

We found experimentally a transition in the shape of the sheared droplet, from an oblate ($\lambda < 1$) to a prolate ($\lambda > 1$) orientation in the vorticity axis.

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